AN IDEA ON P-BRANE AMPLITUDES AND HOW TO COMPUTE THEM

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ABSTRACT. This is a article describing P-brane amplitudes and how to compute them. It is more of a philosophical discussion than a rigorous result. We work in the setting of E-branes, that is P-branes that satisfy the Einstein field equation on the brane sheet. It turned out in an earlier article that if the gravitational anomaly is zero we obtain a diffeomorphism symmetry. We use this symmetry to compute P-brane amplitudes, and state a conjecture related to this. The conjecture is that E-brane diagrams are QFT diagrams, and we argue that this is not the case in general, but could be so in some cases in brane sheet dimension 4 or less. Of course dimension 4 is a very interesting dimension in physics.

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1. A retrospect to th a earlier article on E-branes.

In the previous article 'On Existance of a Large Symmetry Group for Non-Linear Sigma Models and a Self-Consistency Condition for P-Branes' [7], published in Advanced Studies in Theoretical Physics, we gave the following defintions and results;

Assume we are working with a non-linear sigma model or P-brane and have the definition

$$HX^m = \Box^{(p+1)} X^m = \frac{1}{\sqrt{G}} \partial_\mu (G^{\mu\nu} \sqrt{G} \partial_\nu) X^m$$

on our P-branes and non-linear sigma models. μ, ν are indices who live on our brane. m is a index on the target space. So X^m lives on the target space and X^μ lives on the P-brane.

Definition 1.1. *We call a P-brane a E-Brane if it is Einstein with fixed and well-defined stress energy T(For example T must be a cocycle and differentiable on a smooth enough manifold.).The following condition must hold;*

$$\nabla^* (Ric_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) dx^\mu \otimes dx^\nu = 0$$

Which basically demands that the Einstein tensor is a cocycle modulo a affine deformation term in the connection, so we have a statement of cohomology. ∇^* is the covariant divergence. Note that albeit this is true on general position in differential geometry from a contracted Bianchi identity, there may be obstructions making the above expression ill-defined. E-brane dynamics are invariant under the diffeomorphism group, so have a diffeomorphism symmetry then.

Definition 1.2. *A brane is Newtonian with respect to $G_{\mu\nu}$ and X^m if*

$$HX^m = \Box^{(p+1)} X^m = 0$$

for some reference metric $G_{\mu\nu}$ and X^m a field with values in the target space.

Remark 1.1. *We will show that the E-brane Hamiltonian H is invariant under the diffeomorphism group.*

We state the following; If X^m is Newtonian on the P-brane and $G_{\mu\nu}$ is Einstein on our P-brane, then, to first order pertubation $\delta G_{\mu\nu}$ in the metric, the equation

$$\Box^{(p+1)} X^m = \frac{1}{\sqrt{G}} \partial_\mu (G^{\mu\nu} \sqrt{G} \partial_\nu) X^m = 0$$

is satisfied. Expand the above equation times $\sqrt{|G|}$ in a perturbation of the p-brane metric($G_{\mu\nu}$) $\delta G_{\mu\nu}$. Then you get;

$$\sqrt{|G_0|} \Box_0^{(p+1)} X^m + \delta \sqrt{|G|} \Box^{(p+1)} X^m =$$
$$\sqrt{|G_0|} \Box_0^{(p+1)} X^m + \partial_\mu (\sqrt{|G_0|}) (\delta G^{\mu\nu} - G_0^{\mu\nu} \tfrac{1}{2} Tr([G_0][\delta G^{\mu\nu}])) \partial_\nu X^m + \cdots$$

Now, we wish to express $\delta G^{\mu\nu}$ in terms of geometric invariants. Some thought tells us that that the only way to do this is by setting $\delta G_{\mu\nu} = \epsilon Ric_{\mu\nu}$, where ϵ is a small constant(actually a small nice scalar function would be better). Actually we have for the string theory dynamics of the metric

$$\delta G_{\mu\nu} = \beta^G_{\mu\nu} = \alpha' \Delta t Ric_{\mu\nu}$$

where $\beta^G_{\mu\nu}$ is the beta-function and α' is the slope. *This formula is called a Ricci flow.* This is the above forumla with

$$\epsilon = \alpha' \Delta t.$$

So then, on-shell, we obtain

$$= \sqrt{|G_0|}\Box_0^{(p+1)} X^m + \partial_\mu(\sqrt{|G_0|})(\delta G^{\mu\nu} - G_0^{\mu\nu}\tfrac{1}{2}Tr([G_0][\delta G^{\mu\nu}]))\partial_\nu X^m + \cdots =$$
$$\sqrt{|G_0|}\Box_0^{(p+1)} X^m + \partial_\mu(\sqrt{|G_0|})(Ric^{\mu\nu} - \tfrac{Ricci}{2}G_0^{\mu\nu})\partial_\nu X^m \epsilon + \cdots =$$
$$\sqrt{|G_0|}\Box_0^{(p+1)} X^m + \sqrt{|G_0|}\nabla_\mu(8\pi T^{\mu\nu})\partial_\nu X^m \epsilon + \cdots$$

Where the second term should be interpreted as the covariant divergence, and T is some fixed stress-energy T, which of course is conserved at the quantum level if there is no anomaly. So that term vanishes and the Hamiltonian above is invariant under small diffeomorphisms, actually then the entire connected group of the identity $Diff_0(M)$ of the diffeomorphism group $Diff(M)$, M our brane.

2. How to compute amplitudes for E-branes, first approach

Let us start with open branes at tree level. Since you have diffeomorphism symmetry of the E-brane Hamiltonian you just choose a arbitrary coordinate system such as the ball B^N in dimension N, with it's spherical boundary, which you prescribe Dirichlet data on after a Wick rotation. This is exactly the same trick as in solving the harmonic Dirichlet problem on arbitrary connected domains in the complex plane $\mathbb{C}$, we have just found the key to generalize this procedure to higher dimension.

3. A motivation for how to compute ampitudes for E-branes, second approach

Look on any brane diagram of any topology. By standard results from manifold theory we can obtain any compact manifold by surgery on the N-dimensional sphere S^N by gluing tubes with this sphere. Now, looking at the tube (Please note that there are other kinds of tubes necessary to generate all compact manifolds, the tubes below are just one special case.)

$$[0,1] \times S^{N-1}$$

We see it is homotopic to the retract

$$[0,1] \times *$$

where $*$ is a point. The trick is then to use this deformation, which is infitisimally close to the orbit of the diffeomorphism group or in a limit of the diffeomorphisms of this brane since we know that our E-brane Hamiltonian is invariant under diffeomorphisms. Very reminicent to various tricks in residue theory we can deform the integral of any such matrix element to a point in the codimension of the interval above. A point times an interval is what we have left.

Out of the surgery above then there only remains something that looks like QFT diagrams, then not counting on moduli, which should be trivial. It is known from 4-manifold theory that the above story with stitching along QFT diagrams-which is linked to homotopy groups- indeed is fundamental to the topology of 4-folds. There has been much mathematical work on this. Also dimension 3 has been investigated (The Poincare conjecture proved in 2002 and 2003 by Perelman is related to this 'there is something in the fundamental group reasoning'. It is interesting to see that we use Ricci flows to obtain our simplification of diagrams because Perelman also used Ricci flows in his proof of the Poincare conjecture on the 3-sphere.), and dimension 2 is trivial.

4. Could it be that E-brane diagrams are QFT diagrams in disguise?

Maybe. Let us start with open branes. Naively we would compactify them, assuming that we have nice ends of some sorts. So, we are basically taking points on a brane and call them infinity, partially with the motivation that the PDE's we look upon vanish at infinity. If you have a coordinate system, and some possibility of solving the Dirchlet problem in that topology you are done. [1]

Let us restrict to vacuum bubbles in QFT in dimension four. They do define closed 4-manifolds by results by Jones in knot theory. Starting by blowing up the point in the tube just very little you get a closed 4-manifold from your QFT diagram. You can use an embedding to see this, and those always exist by Whitney's theorem. This is possible in any dimension.

A closed manifold with punctures is probably what should concern us. We just showed a naive argument on how to go from QFT diagrams to manifolds. Is the reverse always possible? Is a E-brane diagram a QFT diagram? We believe very solidly in the contrary, since manifold surgery requires other kinds of tubes, of more low-dimensional type. That means that the standard perspective on branes with a co-dimensional time parameter, as in the nomenclature of p-branes, is not good for the general situation in brane theory. However, by results in knot theory and algebraic topology in dimension 4 this could be true in some cases in total brane sheet dimension 4.

To be more specific, a possible way to formulate the problem is to ask what the relation between brane topology, homeomorphism type of branes, diffeomorphism type of branes and the fundamental group π_1 of a brane is. I think that we need to understand this for future developments in brane theory. In dimension 4 many of the results known demand a simplifying assumption.

5. A partial result on amplitudes for E-branes

As stated before just use a good coordinate system or geometry, and then you are done if you know how to solve the harmonic Dirichlet problem on that geometry.

[1] Not necessarily a Cauchy problem. Physicists usually deform the d'Alembertian to an elliptic problem.

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