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A BRIEF REMARK ON NEW FORMULAE FOR STRING AND BRANE THEORY CORRELATORS

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Abstract. This is a article describing String Theory correlators, more specifically Green's functions on compact Riemann surfaces of any genus. The calculus becomes much simpler, without necessary reference to Eistenstein functions or similar formulae in terms of theta-functions. We also grasp towards brane theory correlators, brane Greens functions and brane Sezgo kernals.

1. New Formulae for String and Brane Theory Correlators

This paper will be very brief. Firstly I state that the Laplacian problem on a compact Riemann surface is readily obtainable from the parabolic heat problem. Then, I conjecture the Green's function of the heat problem on a compact Riemann surface of genus g to be given by

$$G = G_+G_-$$

$$G_\pm = \sum_{n\in\mathbb{Z}^g} e^{-n^t\Omega_\pm ns} e^{2\pi i n^t(x_\pm - x'_\pm)}$$

where $x_\pm, x'_\pm$, are of dimension g, $x_\pm \in [0,1]^g$, $x'_\pm \in [0,1]^g$, where g is the genus. Ω is the Schottky matrix, also called the moduli matrix, in conventions such that $Re(\Omega) > 0$ and $s \in [0, +\infty)$ both a time and Schottky parameter.

Noticing the formula

$$\frac{1}{\Delta} = \int_0^\infty e^{-\Delta s} ds$$

where

$$\Delta = -\sum_i \partial_i^2$$

we get

$$D = \frac{1}{\Delta} = \int_0^\infty e^{-\Delta s} ds = \int_0^\infty G ds$$

Performing the integral one obtains the result;

$$D = \sum_{(n,m)\in\mathbb{Z}^{2g}} \frac{1}{(n,m)^t(\Omega_+ \oplus \Omega_-)(n,m)} e^{2\pi i (n,m)^t(x_+ - x'_+, x_- - x'_-)}$$

Then the Sezgo kernal is then given by

$$S_\mp = \partial_\pm D$$

Now, this gives a lot of elliptic function theory, for example the Eisenstein function E would be approximately given by

$$E[\Omega] = E(x - x')[\Omega] \sim e^{2\pi D}$$

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These expressions have the advantage that they generalize to branes quite easily, but we may doubt their validity. Those formulae look like, in dimension m, and with p number of patches,

$$D = \sum_{(n_1,\cdots,n_m)\in\mathbb{Z}^{pm}} \frac{1}{(n_1,\cdots,n_m)^t(\Omega)(n_1,\cdots,n_m)} e^{2\pi i (n_1,\cdots,n_m)^t(x_1,\cdots,x_m)},$$

where $x_i \in [0,1]^p$. We recognize immediately that even this is a quite general function that occurs in Eisenstein theory. The Dirac or Sezgo kernal is then given by

$$S = \frac{1}{\not\partial} = \not\partial\ \frac{1}{\not\partial^{\,2}} = \not\partial\ \frac{1}{\Delta} = \not\partial\ D$$

Then understood that $\not\partial$ denotes the realization of the Dirac operator on each patch of the p number of patches or charts. If you now use Wick's theorem on correlators with the above propagators you are done.

Remark 1.1. *For Einstein branes, also called E-branes, we can always choose a local diffeomorphism, basically a coordinate chart or gauge homotopic to the identity, that that makes the Laplacian a constant coefficient operator, just like for Riemann surfaces* [1]. *An Einstein brane is a brane that follows the Einstein field equation with source term and has no gravitational anomaly. This is not necessarily a black brane, it's more general. This makes the above heat kernels, Green functions and Sezgo kernals work. Basically E-branes have a very simplifying situation. See Torbrand-Dhrif[7]. The MCG(mapping class group) can then be quotiented away just like the case for strings, except we now work with branes.*

2. Topological transitions

We note that the manifold above need not be connected, so it's Hilbert space is a direct sum of the L^2 Hilbert spaces of the appropriate connected parts or manifolds. This means that the above formula decribes a totality of topological dynamics between independent branes of fixed topology. Now, the S-matrix of such a collection of branes with different topologies need be unitary over all channels, so that it preserves probability on the total Hilbert space of the S-matrix. That's like saying that there is a probability of 1 that something will be an out state. I think, but I am not sure, that the description above takes care of the entire E-brane setting of brane dynamics. To be clear; I think that the above may give a simpler and correct representation of correlators on brane sheets for the specific case of Einstein branes, but I am not sure.

References

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[1]Actually we can often just come arbitrarily close, this is a matter of the general topology of the relevant local diffemorphism group or set of coordinate maps. Sometimes we just work in a dense subspace.

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